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Reg. No.

IV Semester B.A. / B.Sc. Degree Examination, September - 2021

MATHEMATICS

(CBCS Scheme Freshers and Repeaters 2015-16 and Onwards)

Paper : IV

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1) Answer **ALL** parts.

PART - A

Answer any **FIVE** questions. **(5×2=10)**

1. a) Define Normal Subgroup of a Group.

b) If $f : G \to G'$ is a homomorphism, then prove that $f(e) = e'$, where e and e' are the identify elements of G and G' respectively.

c) Expand $f(x) = x$ in half range cosine series over the interval $(0, \pi)$.

d) Show that $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at (1,1).

e) If $L[f(t)] = F(s)$, then show that $L[e^{at} f(t)] = F(s-a)$.

f) Find $L[t.\sin t]$

g) Find the particular Integral of $(D^2+1)y = \sin 3x$

h) Verify whether $(1-x^2)y'' - 3xy' - y = 0$ is exact.

PART - B

Answer **ONE FULL** question. **(1×15=15)**

2. a) Prove that a subgroup H of a group G is normal subgroup of G if and only if $gHg^{-1} = H \forall g \in G$.

b) Define centre of a group and prove that the centre of a group G is a normal subgroup of G.

[P.T.O.

c) If $f : G \to G'$ be a homomorphism from the group G into G' with Kernel 'K', then prove that 'f' is one - one if and only if $K = \{e\}$, where 'e' is the identity element in G

(OR)

3. a) Prove that the intersection of two normal subgroups of a group is a normal subgroup.

b) If G is a group and H is a sub-group of index 2 in G, then show that H is a normal subgroup of G.

c) State and prove Fundamental theorem of Homomorphism.

PART - C

Answer **TWO FULL** questions. (2×15=30)

4. a) Find the fourier expansion of in $f(x) = x^2$ (-1,1).

b) Obtain half range sine series of $f(x) = \sin x, 0 < x < \pi$.

c) Expand $x^2y + 3y - 2$ in powers of (x - 1) and (y + 2) by Taylor series upto 3rd degree terms.

(OR)

5. a) Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$.

b) Find the extreme values of the function $f(x) = x^3y^2(1 - x - y)$.

c) Show that minimum value of $x^2 + y^2 + z^2$ subject to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ is 27

6. a) Find $L\left[e^t \sin^2 t\right]$ and $L\left[\sinh 2t - \sin t\right]$.

b) Find $L\left[f(t)\right]$ if $f(t) = \begin{cases} 2t & ; \quad 0 \le t \le 5 \\ 1 & ; \quad t > 5 \end{cases}$

c) Using convolution Theorem find, $L^{-1}\left[\frac{1}{(s+2)(s+4)}\right]$

(OR)

7. a) Find: i) $L\left[t^3e^{-3t}\right]$

ii) $L\left[e^{-t}\left(2\cos 5t - 3\sin 5t\right)\right]$

b) Find $L^{-1}\left[\dfrac{s^2}{(s-1)(s^2+1)}\right]$

c) Find $L\left[t^2u(t-3)\right]$ using convolution property.

PART - D

Answer **ONE FULL** question. **(1×15=15)**

8. a) Solve $\left(D^2 - 2D + 1\right)y = \sinh(x)$.

b) Solve $x^2y_2 - 2x(x+1)y_1 - 2(x+1)y = x^3$, given that x is a part of complementary function.

c) Solve $\left(D^2 + 2D + 4\right)y = e^x \sin x$.

(OR)

9. a) Solve $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = \sin(\text{Log } x)$.

b) Solve $\dfrac{dx}{dt} = 3x - 4y,\ \dfrac{dy}{dt} = x - y$.

c) Solve $y'' + y = \tan x$ by the method of variation of parameters.